



# **Acoustic localization and decay of the incoherent impulse response**

**Final Technical Report  
for ONR Grant No. N00014-98-1-0118,  
titled "Acoustic Localization in Macroscopically Non-uniform Media"  
1 November 1997 - 31 October 1998**

by Eric Smith  
Applied Research Laboratories, The University of Texas at Austin

## **Abstract**

Applied Research Laboratories, The University of Texas at Austin (ARL:UT) was tasked to study the effects of strong random scattering in acoustic media with macroscopically non-uniform scattering strengths. A statistical, field-theoretic method, previously applied in condensed matter physics and seismology, was introduced to study these effects analytically. Non-uniform random scattering was found to produce phenomena closely related to acoustic localization, which results from uniform scatter. However, non-uniformity also introduces new qualitative signatures not found in localizing systems, including exponential decay of the incoherent impulse response, with universal scaling of the decay time constant. These predictions match features long known empirically to characterize earthquake coda, but not previously explained. The seismic data thus validate the new mathematical techniques, and also introduce useful ways to characterize random scattering within sediments.

## **Objectives**

The study of acoustic reflection and propagation within saturated sediments, even within the smooth-medium approximation, involves two fundamental problems. The first is identifying the range of qualitative effects intrinsically associated with scattering in randomly-composed media. The second is choosing the correct representation of real sediments: effective fluid, elastic solid, Biot medium, or beyond. FY 1998 tasks were intended to address the first problem, in particular the analysis of qualitative signatures associated with strong, generally high-order scattering.

Among the non-obvious consequences of strong random scattering in statistically uniform media is the energy-trapping phenomenon known as *acoustic localization*.<sup>1</sup> Treatment of uniform localization by numerical simulation has proven difficult in more than one dimension, though, and apart from a specific model of seismic reflection in a randomly layered halfspace, non-uniform

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scattering strength has not been considered.<sup>2</sup> Therefore, the center of FY 1998 work was the development of new analytic methods to predict the statistical features of such scattering in arbitrary dimensions.

Only fluid media were to be studied in this task, with the intent to apply the results to elastic, poro-acoustic, and eventually Biot models in succeeding years. The problem of smooth non-uniformity of a statistically random scatterer distribution was considered, both as a simple first example, and as the most general, leading-order departure from the well-understood phenomenon of uniform localization.

Because of the difficulty of assigning physical origins to incoherent signal features, it was important that any new methods introduced predict well-established and distinctive signatures of data attributed to random scattering. The best such constraint was provided by earthquake seismology: band-passed earthquake coda (the rumble after the direct arrival of a shock) is known to decay exponentially, with the time constant of the decay scaling as the one-third power of frequency. This observation is universal, appearing to characterize earthquakes and explosions from all around the world, under widely varying propagation conditions.<sup>3</sup> Almost all of its features - later persistence of high-frequency energy, time constant scaling as a non-integer power of frequency, and universal behavior - suggest a multiple-scattering origin, but are not predicted by any previous models.

The objectives for FY 1998 were to construct a mathematical framework to treat non-uniform, high-order random scattering, and to show that such scattering accounts *naturally* for the special scaling behaviors of seismic coda, not depending on unphysically fine tuning of model details. The framework was also intended to show the relation of such features as high-frequency persistence of energy and universality to localization in the limiting, statistically uniform case.

## Results

A field-theoretic method, previously used in condensed matter physics to predict both acoustic localization and its electronic counterpart, was successfully generalized to describe the effects of non-uniform random scattering.<sup>4</sup> The derivation was carried out for general dimensions, so it can describe scattering from layer randomness, linear features, or the fully three-dimensional randomness of a sediment frame or the earth's lithosphere.

New qualitative signatures were shown to be created by non-uniform random scattering. The uniform scattering that leads to traditional localization

yields a prediction that nearly-trapped energy decays according to a diffusion equation,<sup>1</sup> which does not match the exponential decay found for seismic coda.<sup>3</sup> Non-uniform scattering was shown to replace the trapped modes of localization with nearly-trapped modes, which decay exponentially in the manner of incoherent resonances, providing a very compelling explanation of the seismic data. The nature of the derivation shows that the formation of such resonances is closely related to the formation of localized modes, in particular, that they arise from very high-order multiple scattering, and are not approximated by small-scatterer perturbation theory.

The new derivation provides, for the first time, an explanation for the universal scaling of the decay-time constant observed in seismic coda. It shows that such universal scaling arises when the acoustic wavelength is much larger than all relevant features of the scatterers, and is controlled only by the dimension of the space in which the scattered waves propagate (planar, cylindrical, or spherical). For one-dimensional scattering, as arises in a randomly layered medium, it predicts a time constant scaling as the one-third power of frequency, again matching data from earthquake coda. The scaling relation is more complicated in higher dimensions, but is expected to be qualitatively similar.

The derivation of the effects of non-uniform scattering also remedies an outstanding deficiency in the mathematical theory of uniform acoustic localization, which is of interest in condensed matter as well as sediment acoustics. Though it was known that the formation of localized modes could not be derived with perturbation theory, it had not been shown how to complete such perturbation theory in calculations of acoustic localization. The current derivation recovers uniform localization as a limiting case, but shows that small, finite non-uniformity can be used as a regulator. The regulated calculation makes clear how to complete the uniform-limit perturbation theory, and also shows that the smoothly-varying model chosen indeed predicts the first departures from uniform localization.

## **Outlook**

The statistical framework, constructed to describe non-uniform random scattering in FY 1998, has immediate generalizations to elastic and multi-component (e.g., Biot) media. The universal nature of the qualitative signatures it predicts strongly suggests that these features will have counterparts in the more complex systems.

The ability of these methods to predict statistical regularities of incoherent signals, with the empirical analysis of seismic coda serving as an example, introduces new methods for analysis of reflection and propagation data in sediments. At the same time, the constraint of a long-known scaling behavior in a very comprehensive data set shows, in concrete terms, how the field-theoretic methods may be used and interpreted.

### **Publication**

E. Smith, "Localization in non-uniform media: Exponential decay of the late-time Ginzburg-Landau impulse response," Phys. Rev. B58 (1998) 5346-5366

### **Published abstract**

E. Smith, "Acoustic localization and decay of the incoherent impulse response," J. Acoust. Soc. Am. 104(3) (1998) 1809 (A)

### **References**

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APPLIED RESEARCH LABORATORIES  
THE UNIVERSITY OF TEXAS AT AUSTIN

P. O. Box 8029 • Austin, Texas 78712-8029 • (512) 835-3200 • FAX: (512) 835-3259

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From: Advanced Sonar Division

To: Director  
Office of Naval Research  
Ballston Tower One  
800 North Quincy Street  
Arlington, VA 22217-5000

Attn: Dr. Jeffrey Simmen, ONR 321OA

Subj: Final Technical Letter Report

Encl: (1) Final technical report titled, "Acoustic localization and decay of the incoherent impulse response"  
(2) Copy of Material Inspection and Receiving Report (DD form 250), ATL0407

Ref: (a) Contract N00014-98-1-0118 titled, "Acoustic Localization in Macroscopically Non-Uniform Media," 1 November 1997 through 31 October 1998

1. Under Ref. (a), Applied Research Laboratories, The University Texas at Austin (ARL:UT), was tasked to study the effects of strong random scattering in acoustic media with macroscopically non-uniform scattering strengths.

3. Enclosure (2) is forwarded as required by DFARS, Appendix I, Distribution for the Material Inspection and Receiving Report. A signed DD Form 250 is necessary to document the delivery of contractually required items. Please sign and return one copy to the ARL:UT address shown above, marked to the attention of the Contracts Office.

3. If you have any questions, please contact me via email at [desmith@arlut.utexas.edu](mailto:desmith@arlut.utexas.edu).

*Eric Smith by B Jones*  
Eric Smith

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